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Effect of Variable Lewis Number on Heat Transfer in a Binary Gas

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Introduction

SEVERAL authors have implicitly considered the effect of variable Lewis number in a binary diffusion process, particularly for the case of local equilibrium in a stagnation-point boundary layer [1–4] Anderson[4] found that the stagnation-point heat-transfer rate was predicted within 5% when the average Lewis number in the boundary layer was used in the Fay and Riddell formula In this note, the effect of variable Lewis number is explicitly determined for the equilibrium and frozen Couette flow of an ideal dissociating diatomic gas The results[5] indicate that the constant Lewis number giving the same heat-transfer rate as the variable Lewis number solution may be simply expressed in terms of the freestream and wall values of Lewis number and is only weakly dependent on freestream velocity and wall temperature

Analysis

The equations of state, enthalpy, and equilibrium atom mass fraction for an ideal dissociating diatomic gas are (in the notation of Clarke[6])

$$p = (1 + Ca)\rho R_0 T/W_m \quad (1)$$

$$H = (4 + Ca)R_0T/W_m + CaD \quad (2)$$

$$Ca = [1 + (pW_m/\rho_d R_0 T)\exp(W_m D/R_0 T)]^{-1/2} \quad (3)$$

The gas transport properties are based on Moore,[7] who has shown that, to a good approximation, the ratio of mixture viscosity to molecular viscosity and the reciprocal Schmidt number $1/Sc = Le/Pr$ of a binary mixture vary linearly with atom mole fraction In terms of atom mass fraction, the viscosity and Lewis number become

$$\mu = C_2 T^{0\,75}(1 + 1\,234Ca)/(1 + Ca) \quad (4)$$

$$Le = (C_3 Pr)(1 - 0\,234Ca)/(1 + Ca) \quad (5)$$

$Pr = 0\,74$ and $C_3Pr = 1\,45$ were used in the analysis, since the Prandtl number variations are quite small compared with those of Lewis number

Integration of the momentum and energy equations for a Couette flow leads to the following results:

Shear

$$\tau = \mu(du/dy) = \text{const} \quad (6a)$$

Velocity Profile

$$\frac{y}{\delta} = \frac{2}{Re_\delta C_f}\int_0^{u/u_\delta}\frac{\mu}{\mu_\delta}\,d\left(\frac{u}{u_\delta}\right) \text{ where } Re_\delta = \frac{\rho_\delta u_\delta \delta}{\mu_\delta} \quad (6b)$$

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Enthalpy Profile

$$H - H_w + D[L(Ca) - L(Ca_w)] + (Pr/2)u^2 = -q(Pr/\tau)u \quad (6c)$$

where $L(Ca) = (Le - 1)Ca$ for constant Le and $= 1\,789\ln(1 + Ca) - 1\,339Ca$ for variable Le q is the heat flux rate, D is the dissociation energy per unit mass of atoms, and the subscripts w and δ refer to the boundary values at the surface $y = 0$, where $u = 0$, and at the outer edge of the parallel viscous flow, where $y = \delta$, $u = u_\delta$ From Eqs (6) we obtain the following values for the skin-friction and heat-transfer coefficients:

$$C_f = \frac{2\tau}{\rho_\delta u_\delta^2} = \frac{2}{Re_\delta}\int_0^1 \frac{\mu}{\mu_\delta}\,d\left(\frac{u}{u_\delta}\right) \quad (7)$$

$$St = -q/\rho_\delta u_\delta(H - H_w) = (C_f/2Pr)[1 + D\{L(Ca_\delta) - L(Ca\ )\}/(H - H_w)] \quad (8)$$

The recovery enthalpy H_r is the value of H_w when $q = 0$ Using Eqs (6), we obtain the enthalpy recovery factor:

$$\theta = \frac{H_r - H_\delta}{H_{t\,t\,1} - H_\delta} = Pr + \frac{2D}{u_\delta^2}[L(Ca_\delta) - L(Ca\ )] \quad (9)$$

The solution is now complete except for the determination of the atom concentration in the layer For equilibrium flow, $Ca = Ca$ [Eq (3)] For frozen flow, the continuity of species equation may be written as

$$\frac{d^2Ca}{du^2} - \frac{1}{Sc}\frac{dSc}{dCa}\left(\frac{dCa}{du}\right)^2 = 0 \quad (10)$$

which, with Eq (5), is integrated to give the atom concentration in the layer as

Variable Le

$$\frac{u}{u_\delta} = \frac{1\,234\ln[(1 + Ca)/(1 + Ca_w)] - 0\,234(Ca - Ca_w)}{1\,234\ln[(1 + Ca_\delta)/(1 + Ca_w)] - 0\,234(Ca_\delta - Ca_w)} \quad (11a)$$

Constant Le

$$Ca = Ca_w + (Ca_\delta - Ca_w)u/u_\delta \quad (11b)$$

The freestream atom concentration Ca_δ was arbitrarily assumed to correspond to equilibrium The wall atom concentration Ca_w is determined by the equation $(\rho DamdCa/dy)_w = \Gamma S$, where S is the atom mass flux rate per unit area to the wall, and Γ is the fraction of these atoms which recombine Assuming that the atoms near the wall have a Maxwellian velocity distribution, we find

$$S = \left(\frac{Ca_w}{1 + Ca_w}\right)\frac{p}{(\pi R_0 T_w/W_m)^{1/2}}$$

Using these results in Eqs (11) then gives the following ex pressions for the wall atom concentration:

Variable Le

$$1\,234\ln\left(\frac{1 + Ca_\delta}{1 + Ca_w}\right) - 0\,234(Ca_\delta - Ca_w) = \frac{0\,5103u_\delta}{\tau_w}\left(\frac{Ca_w}{1 + Ca_w}\right)\frac{\Gamma p}{(\pi R_0 T_w/W_m)^{1/2}} \quad (12a)$$

Constant Le

$$Ca_w = \tfrac{1}{2}[b + (b^2 + 4Ca_\delta)^{1/2}]$$

where

$$b = Ca_\delta - 1\,\frac{-\,Sc_w\Gamma u_\delta p}{\tau_w(\pi R_0 T_w/Wm)^{1/2}} \quad (12b)$$

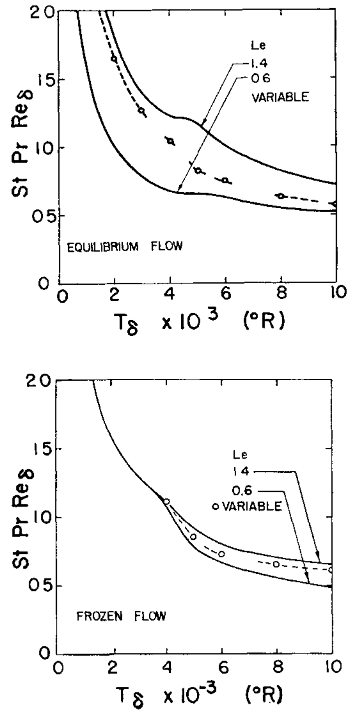

Fig 1 Effect of Lewis number on heat transfer (oxygen, $u_\delta = 15{,}000$ fps, $p = 10^{-3}$ atm, $T_w = 1000$ R, $\delta = 10^{-2}$ ft, fully catalytic wall)

Results and Discussion

The effects of variable Lewis number was evaluated for oxygen by comparing constant and variable Lewis number results for equilibrium flow and for frozen flow with a fully catalytic wall ($\Gamma = 1$) Note that, for a frozen flow with a noncatalytic surface ($\Gamma = 0$), Eqs (12) show that $Ca_w = Ca_\delta$, and Le = const = Le_δ in the layer, so that this case need not be considered further

The value of Lewis number was found to have very little influence on the velocity profiles or skin-friction coefficient, a minor effect on the temperature profiles, and a marked effect on atom concentration and heat transfer [5]

The effect of Lewis number on the heat-transfer parameter $StPrRe\delta$ is shown as a function of freestream temperature for a freestream velocity of 15,000 fps in Fig 1 In the equilibrium case, Lewis number has a significant effect on heat transfer even at low freestream temperatures, since the middle of the layer is dissociated In frozen flow there is no effect at temperatures at which the freestream is undissociated, since $Ca \leq Ca_\delta$

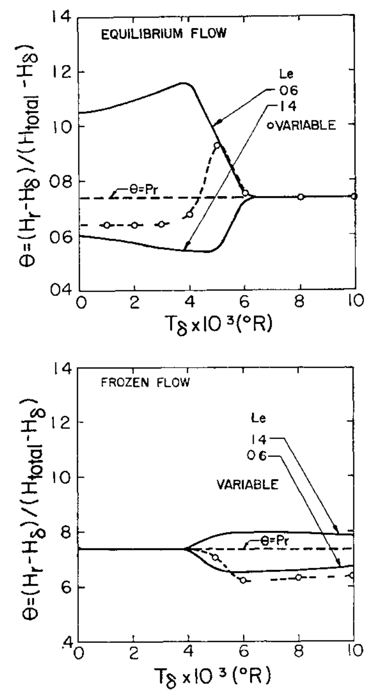

Fig 2 Effect of Lewis number on enthalpy recovery factor (same flow conditions as in Fig 1)

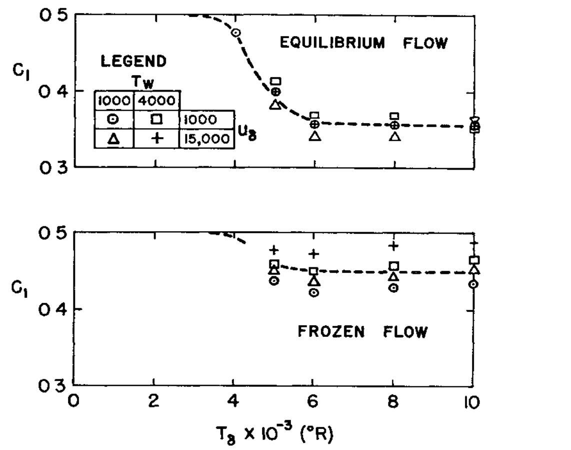

Fig 3 Weighting factor C_1 in the equation $\overline{Le} = C_1Le_w + (1 - C_1)Le_\delta$

To calculate heat-transfer rates, it is also necessary to know the value of the enthalpy recovery factor θ (Fig 2) $\theta = Pr$ at high freestream temperatures in equilibrium flow, since $Ca_\delta = Ca = 1$, and at low freestream temperatures in frozen flow because $Ca = Ca_\delta = 0$ [see Eq (9)] It may further be noted that, in equilibrium flow, $\theta > Pr$ when $Le < 1$, since $Ca \geq Ca_\delta$, whereas in frozen flow the opposite is true, since $Ca \leq Ca_\delta$ Under some conditions, $\theta > 1$ for $Le < 1$ in equilibrium flow, since there must be a negative temperature gradient at the surface (i e , a higher enthalpy near the surface than in an undissociated flow) in order for an amount of heat equal to that transported to the surface by diffusion to be carried back into the flow by conduction

Calculations were made to determine what mean Lewis number $\overline{Le}$ would result in the same heat-transfer rate as the variable Lewis number solution Figure 3 shows the weighting factor C_1 in the assumed relationship $\overline{Le} = C_1Le_w + (1 - C_1)Le_\delta$ The figure indicates that C_1 is fairly independent of freestream velocity and wall temperature The dotted curves drawn through the data are well represented by taking C_1 to be a linear function of $Le\delta$ Then, for equilibrium flow,

$$\overline{Le} = 0\,273Le_w + 0\,727Le_\delta + 0\,156Le_\delta(Le_w - Le_\delta) \quad (13a)$$

For frozen flow,

$$\overline{Le} = 0\,419Le_w + 0\,581Le_\delta + 0\,0558Le_\delta(Le_w - Le_\delta) \quad (13b)$$

The appropriate mean Lewis number to be used in heat-transfer calculations thus differs appreciably from the average Lewis number in the boundary layer

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Flow of Second-Order Fluids with Heat Transfer

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THE constitutive equation of an incompressible second-order fluid has been suggested by Coleman and Noll[1] as

$$\tau_{ij} = -p\delta_{ij} + \mu_1 A_{(1)ij} + \mu_2 A_{(2)ij} + \mu_3 A_{(1)ik} A_{(1)kj} \quad (1)$$

where

$$\begin{aligned} A_{(1)ij} &= v_{i\,j} + v_{j\,i} \\ A_{(2)ij} &= a_{i\,j} + a_{j\,i} + 2v_{m\,i}v_{m\,j} \end{aligned} \quad (2)$$

τ_{ij} is the stress tensor, v_i and a_i are the velocity and the acceleration vectors, respectively, μ_1, μ_2, and μ_3 are the material constants, and p is an indeterminate hydrostatic pressure The equation of motion of this fluid in dimensionless form can be written as

$$\frac{\partial \mathbf{v}}{\partial t} + \boldsymbol{\omega} \times \mathbf{v} + \left(\frac{\mu_1}{\rho L U}\right)(\text{curl}\boldsymbol{\omega}) + \left(\frac{\mu_2}{\rho L^2}\right) \times \left[\frac{\partial}{\partial t}\text{curl}\boldsymbol{\omega} + (\mathbf{v}\ \nabla)\text{curl}\boldsymbol{\omega}\right] + \left[\frac{2(\mu_2+\mu_3)}{\rho L^2}\right]\left[\frac{\partial \mathbf{v}}{\partial x} \times \frac{\partial \boldsymbol{\omega}}{\partial x} + \frac{\partial \mathbf{v}}{\partial y} \times \frac{\partial \boldsymbol{\omega}}{\partial y} + \frac{\partial \mathbf{v}}{\partial z} \times \frac{\partial \boldsymbol{\omega}}{\partial z} + \{(\text{curl}\boldsymbol{\omega})\ \nabla\}\mathbf{v} + \frac{1}{2}(\boldsymbol{\omega}\ \nabla)\boldsymbol{\omega}\right] = -\left(\frac{P_0}{\rho U^2}\right)\nabla(p+s) \quad (3)$$

where

$$2s = \rho|\mathbf{v}|^2 - (6\mu_2 + 4\mu_3) \times \left[\left|\frac{\partial \mathbf{v}}{\partial x}\right|^2 + \left|\frac{\partial \mathbf{v}}{\partial y}\right|^2 + \left|\frac{\partial \mathbf{v}}{\partial z}\right|^2\right] + \mu_2|\boldsymbol{\omega}|^2$$

v and **ω** are velocity and vorticity vectors, respectively, ρ is the density, L is the typical length of the system, and U is the velocity comparable to actual velocities Inspectional analysis reveals that the second-order effect will be important if A or B or both $\gg 1$ ($A = \mu_2 U/\mu_1 L, B = \mu_3 U/\mu_1 L$), which for a particular fluid can be fulfilled either making U large or L small Hence, if such a liquid has to pass through fine gaps or is sheared between two parallel rotating plates (small distance apart), the second-order terms will play a significant part in determining the nature of the flow

Consider the motion of a second-order fluid filling the space between two infinite plates, one of which ($z = 0$) is rotating with constant angular velocity Ω about an axis ($\gamma = 0$) perpendicular to its own plane and the other ($z = d$) is stationary The rotating plate is maintained at a temperature T_0 and the stationary one at T_1 ($T_1 > T_0$) Taking u, v, w to be the velocity components in the directions of γ, θ, z, respectively, in cylindrical polar coordinates, the boundary conditions in this problem can be written as:

$$\begin{aligned} &T = T_0 \quad u = 0 \quad v = \gamma\Omega \quad w = 0 \quad \text{at } z = 0 \\ &T = T_1 \quad u = 0 \quad v = 0 \quad w = 0 \quad \text{at } z = d \end{aligned} \quad (4)$$

The velocity components, the pressure, and the temperature are taken to be of the form

$$u = \gamma\Omega F'(\eta) \qquad v = \gamma\Omega G(\eta) \qquad w = -2d\Omega F(\eta) \quad (5)$$

$$T = T_0 + (\mu_1\Omega/\rho Cp)[\phi(\eta) + (\gamma^2/d^2)\psi(\eta)] \quad (6)$$

$$p = \mu_1\Omega[-p_1 + (\gamma^2/d^2)\{(2A+B)(F''^2 + G'^2) + \lambda\}] \quad (7)$$

where $\eta = z/d$, $A = \mu_2\Omega/\mu_1$, $B = \mu_3\Omega/\mu_1$, Cp is the specific heat at constant pressure and λ is a constant to be determined by the boundary conditions This form of pressure has been suggested by the author in an earlier paper on second-order fluids [2] With this form of velocity components, pressure, and the temperature, the equations of motion in the directions of γ, θ, z, respectively, become

$$R(F'^2 - G^2 - 2FF'') = F''' - 2\alpha R(F''^2 + 2G'^2 + FF^{iv}) - \beta R(F''^2 + 3G'^2 + 2F'F''') - 2\lambda \quad (8)$$

$$2R(F'G - FG') = G'' + 2\alpha R(F''G' - FG''') + 2\beta R(F''G' - F'G'') \quad (9)$$

$$4RFF' = p_1' - 2F'' + 4\alpha R(11F'F'' + FF''') + 28\beta RF'F'' \quad (10)$$

where $\alpha = (\mu_2/d^2)$, $\beta = (\mu_3/d^2)$, and a prime denotes differentiation with respect to η The energy equation consists of terms that are independent of γ and terms containing γ^2 Equating the terms independent of γ and coefficient of γ^2 on both sides of the energy equation we have

$$(RP)^{-1}(4\psi + \phi'') + 12F'^2 + 2F\phi' - 24\alpha FF'F'' - 24(\alpha+\beta)F'^3 = 0 \quad (11)$$

$$(RP)^{-1}\psi'' + F''^2 + G'^2 - 2(\psi F' - F\psi') - 2\alpha F(F''F''' + G'G'') - (2\alpha + 3\beta)F'(F''^2 + G'^2) = 0 \quad (12)$$

where P is the Prandtl number given by $\mu_1 Cp/\rho k$ (k being the conductivity of the fluid)

For small values of R, a regular perturbation scheme for Eqs (8–12) can be developed by expanding F, G, ϕ, ψ, and λ in powers of R Substituting

$$\begin{aligned} F &= F_0 + RF_1 + R^2F_2 + \\ G &= G_0 + RG_1 + R^2G_2 + \\ \phi &= \phi_0 + R\phi_1 + R^2\phi_2 + \\ \psi &= \psi_0 + R\psi_1 + R^2\psi_2 + \\ \lambda &= \lambda_0 + R\lambda_1 + R^2\lambda_2 + \end{aligned}$$

in Eqs (8–12) we get linear differential equations in (F_0, G_0, ϕ_0, ψ_0, λ_0), (F_1, G_1, ϕ_1, ψ_1, λ_1), (F_2, G_2, ϕ_2, ψ_2, λ_2), etc , which have been solved This method of expansion has been adopted by the author in solving similar problems in Reiner-Rivlin fluid[3] as well as in electrically conducting fluid in presence of a transverse magnetic field [4] Neglecting R^3 and higher powers of R, we get the following solution:

$$u = \gamma\Omega R(\tfrac{1}{10}\eta - \tfrac{7}{20}\eta^2 + \tfrac{1}{3}\eta^3 - \tfrac{1}{12}\eta^4) \quad (13)$$

$$v = \gamma\Omega[(1-\eta) + R^2\{-\tfrac{1}{700}\eta + \tfrac{1}{30}\eta^3 - \tfrac{1}{12}\eta^4 + \tfrac{2}{25}\eta^5 - \tfrac{1}{30}\eta^6 + \tfrac{1}{201}\eta^7 + (\alpha+\beta)(\tfrac{1}{10}\eta^2 - \tfrac{7}{30}\eta^3 + \tfrac{1}{6}\eta^4 - \tfrac{1}{30}\eta^5)\}] \quad (14)$$

$$w = -d\Omega R(\tfrac{1}{10}\eta^2 - \tfrac{7}{30}\eta^3 + \tfrac{1}{6}\eta^4 - \tfrac{1}{30}\eta^5) \quad (15)$$

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